

Filtered PWM Digital to Analog Converter

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Abstract

In many digital embedded systems, it is necessary to have Digital to Analog Conversion in order to interface with our analog world. While some microcontrollers have built in features to assist in this matter, many lower cost or low power controllers do not. This document is meant to explain a simple method of using a pulse width modulated square wave in order to create an adjustable voltage analog output.

Digital to Analog Converters

Digital to Analog Converters (DACs) take in some digital input to create an analog output. There are many methods to create DACs. There are many DAC chips on the market designed to interface with microcontrollers. Such chips take input from a microcontroller either in parallel or serial. Some common benchmarks used to compare such chips are resolution, speed, and cost.

The resolution of a DAC is related to the number of steps between the highest output voltage and the lowest output voltage. For example, a 10 bit DAC with a single ended +5V supply would be able to output a voltage with $5V/2^{10} = 4.88$ mV increments. The greater the bit depth of a DAC, the greater the resolution of the output.

Pulse Width Modulation

Pulse Width Modulation (PWM) involves changing the duty cycle of a fixed frequency square wave, whose frequency will be referred to in this document by the constant f_0 . Duty cycle is defined as the ratio of the time that the square wave is at its peak value (t_h) to its period, $T = 1/f_0$. The duty cycle, $D = t_h/T$ is a

finite number with values between 0 and 1. PWM is easily implemented in most microcontrollers and can be utilized for many applications.

Typically, PWM is implemented by utilizing timers in a microcontroller. The timer will increment each clock cycle to a programmer specified value each period. As the timer counts up to the specified value, another section of logic looks to see if the value is beyond some intermediate value, also set by the programmer. This is the time that the wave is at it's high state. As the intermediate value is increased, the time the pulse is high increases, which increases the duty cycle of the square wave.

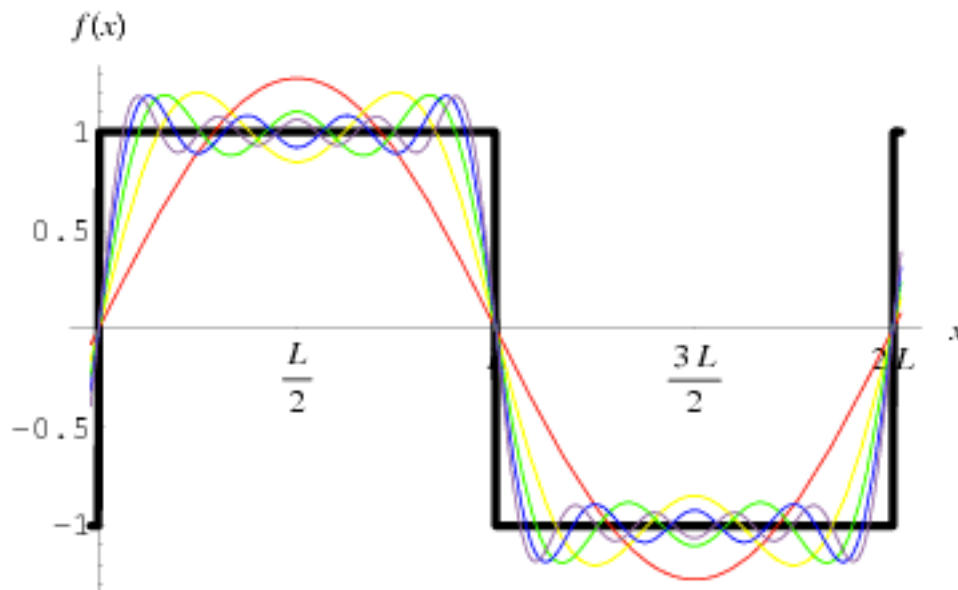


Figure 1

To create an DAC from a PWM signal, the PWM's Fourier series expansion is exploited. The theory behind Fourier series expansion is that any periodic signal can be represented by an infinite sum of sinusoids. This can be seen in Figure 1 above [1]. The first value of a Fourier series expansion is the average value of the signal over it's period, and the other sinusoids occur at multiples of the fundamental frequency, f_0 . The sinusoids of frequency $n \cdot f_0$ are known as harmonics, where n is a positive, non zero, whole number.

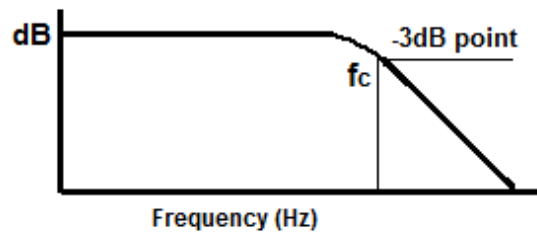
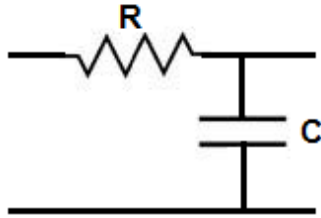
Low Pass Filters

One of the key components of a PWM DAC is the low pass filter. Filters have a 'pass band' of frequencies which will pass through the filter and a 'stop band' of frequencies that will be filtered out. An ideal low pass filter will pass low frequencies un-attenuated in the pass band and completely reject all frequencies in the stop band. Such a filter can not be realized with actual components. Real low pass filters will not completely reject frequencies in the stop band.

Frequency components of the signal to be filtered will pass with minimal attenuation, while Frequency components of a signal in the stop band will be attenuated by a factor that decreases as frequency increases. The slope of this decrease is determined by the number of 'reactive' components. Reactive components have an impedance that changes with frequency.

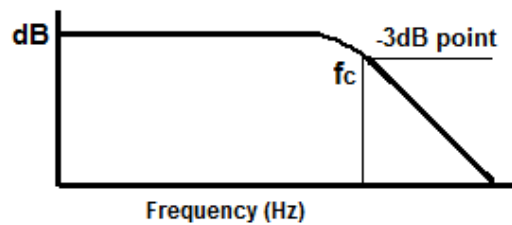
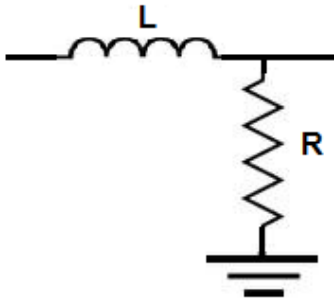
Capacitors and Inductors are used in filters, because their impedance changes linearly with frequency. Perhaps the simplest low pass filter consists of a single capacitor or inductor and a resistor. The frequency denoting the starting of the stop band is denoted by f_c , the cutoff frequency. This is the frequency at which the input signal will be attenuated by -3 dB. Below are implementations of a simple low pass filters with inductors and capacitors [2].

RC Low Pass Filter



$$f_c = \frac{1}{2\pi RC}$$

RL Low Pass Filter



$$f_c = \frac{R}{2\pi L}$$

The 'order' of a filter describes the filter's attenuation in the stop band. Generally this has a direct correlation with the number of reactive components in the filter. The slope of the attenuation of the stop band is the order of the filter multiplied by -20 dB/decade. A third order filter will have 3 reactive components and a stop band attenuation of -60 dB/decade. Thus the higher the order of the filter, the higher attenuation of the stop band.

PWM DAC

In order to create a DAC with a PWM signal, we must low pass filter the signal to pass only the DC component of the waveform. The DC component is the average value of the waveform. As duty cycle increases, the average value of the waveform goes up because the pulse is at its high state for a longer amount of time. In fact, for a perfectly square PWM waveform, the average value of the wave is the duty cycle times the voltage at its peak, assuming the low value is exactly 0V. Thus, with a PWM duty cycle under software control, a simple DAC can be implemented. This DAC requires minimal processor time to change its output value, as there is no communication with another integrated circuit to be implemented. It will also have a pretty good resolution since PWM duty cycle can often be changed by fractions of a percent.

In order to pass only the DC component of the PWM waveform, a low pass filter with a cutoff frequency, f_c much lower than the fundamental frequency of the PWM pulse, f_0 should be designed. The schematic for the design used in Design Team 10's circuit can be seen in Figure 2. The PWM signal is applied to the node labeled V_{in} , and the filter output is seen at V_{out} . In our design it was also necessary to boost the output voltage so its range is from 0 – 10V. To do this, we picked R_2 and R_1 to create a gain of 3, so that the microcontroller's 3.3 v output will have a maximum of around 10V at V_{out} .

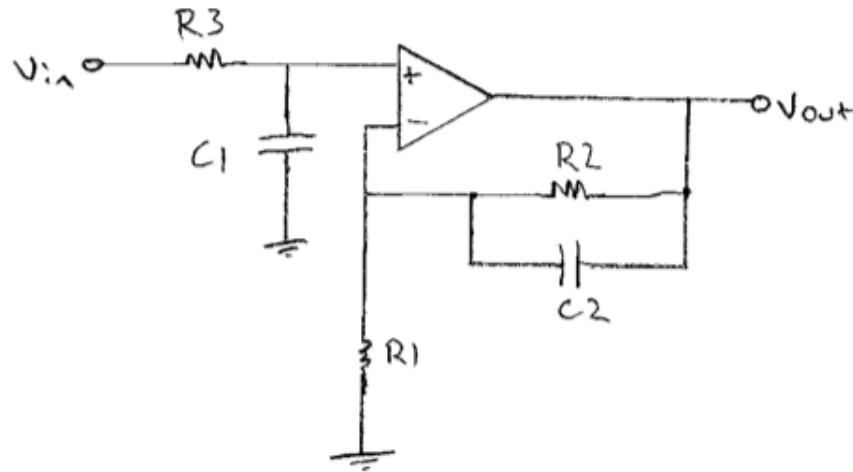


Figure 2

As noted in the discussion of real vs. ideal filters, frequency components of a waveform can not be completely rejected by a filter made out of real components. In this design, a noticeable ripple can be seen if the cutoff frequency of the low pass filter is too close to the fundamental frequency of the PWM pulse. To minimize ripple on the output, f_c of the low pass filter should be chosen to be as low as possible with available components, and the 'order' of the low pass filter should be as high as possible given constraints on physical size and cost of components.

A PWM based DAC only requires a change in duty cycle to change the output voltage. While the output can have significant ripple, this can be filtered out with a higher order filter, and increasing the PWM fundamental frequency.

References

[1] <http://mathworld.wolfram.com/FourierSeriesSquareWave.html> - graph used in document.

[2] <http://www.learningaboutelectronics.com/Articles/Low-pass-filter-calculator.php> - images of filter circuits used in document.